to the thermoelastic problems of thin cylindrical shells. When the shear τ is constant, when the effect due to τ_2 [see Eq. (10)] is neglected, and when the radial deflection is small, the problem becomes the one solved in Ref. 2.

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Comparison of Theory with Experiment on a Blunt Axisymmetric Body in Hypersonic Flow

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Nomenclature

a = nondimensional shock layer thickness parameter
a^* = the reference value of a when the parameter $Re_R/M(C)^{1/2}$ is very large
C_H = surface heat transfer coefficient $q/\rho_\infty u_\infty(H_\infty - H_w)$
C_D = nose drag coefficient
C = Chapman-Rubesin constant
D = diameter of model
H = total specific enthalpy
I = $[0.664 + 1.73(T_w/T_0)]$
K = value of p/p_s as $y/y_s \to 0$
M = freestream Mach number
p = pressure
R = radius of model
Re_R = Reynolds number $- \rho_\infty u_\infty R/\mu_\infty$
T = temperature
t = half the thickness of the nose
u = velocity parallel to model axis of symmetry
x,y = rectangular coordinates in direction parallel and normal, respectively, to the model axis of symmetry, with the origin at the nose shoulder
γ = specific heat ratio
ϵ = $\gamma - 1/\gamma + 1$
μ = viscosity coefficient of the gas
ρ = density

Subscripts

b = edge of the effective wall
e = outer edge of the entropy layer
s = immediately behind the shock wave
0 = stagnation conditions on the body
w = wall conditions
∞ = freestream conditions

Recently there have been two new approaches to the solution of the two-dimensional blunt leading edge problem over a flat plate.[1, 2] The use of these developments to predict the heat transfer over a blunt flat plate has the definite advantage of not requiring an experimentally determined pressure distribution. The extension of these theories to the axisymmetric case and comparison with experiment was presented in Ref. 3. This note is a continuation of Ref. 3 specifically dealing with the extension of Ref. 2 to the axisymmetric case.

Cheng et al.[1] obtained a basic differential equation for y_e in terms of the nose drag coefficient for both the two-dimensional and axisymmetric cases valid in the limit of small ϵ. A solution was obtained for the two-dimensional case only. The solution of the axisymmetric problem was obtained in Ref. 3. Once y_e is known, the pressure and heat transfer distributions can be determined by

$$p_w/p_\infty = \gamma M^2[y_e'^2 + (y_e y_e''/2)]$$

$$M^3 C_H = 0.332\, M^3(C)^{1/2}/(Re_R)^{1/2}(p_w/p_\infty) \times \left[\int_0^{x/R} p_w/p_\infty d(x/R)\right]^{-1/2}$$

Oguchi's two-dimensional theory[2] can be extended to the axisymmetric case in the following manner. The basic equation in Oguchi's paper (Eq. 3.4) relating the entropy layer thickness to the pressure in the entropy layer can be obtained by writing an approximate mass balance in the entropy layer:

$$\int_0^{y_e} \rho u dy \approx \rho_\infty u_\infty t$$

Assuming that the pressure is constant through the entropy layer and the velocity is approximately equal to the freestream velocity (which was also done in Oguchi's analysis) the forementioned equation can be reduced to

$$y_e = (p_0/p_e)^{1/\gamma}\, \epsilon t$$

which is the same expression obtained by Oguchi. Proceeding in the same manner for an axisymmetric body of constant radius the corresponding relation is

$$y_e^2 = (p_0/p_e)^{1/\gamma}\, \epsilon R^2$$

provided that R^2 can be neglected compared to y_e^2. For an axisymmetric body the shock shape is given by

$$y_s/2R = a(x/2R)^{1/2}$$

Using the hypersonic small disturbance theory result that

$$p_s/p_0 = (1/K)(p_e/p_0)$$

and a pressure law of the form

$$(dy_s/dx)^2 = p_s/p_0$$

the forementioned relations can be combined to give the pressure on the body. It was assumed that the boundary layer displacement correction obtained in Ref. 2 for the two-dimensional case also applies to the axisymmetric case by the use of Mangler's transformation. Adding this in the resulting expression for the pressure is

$$p_w/p_\infty = [\gamma/2(\gamma + 1)]Ka^2M^2(x/2R)^{-1}$$

where

$$a = a^*\left[1 + \delta a^{*[(2-\gamma)/2]} \frac{M(C)^{1/2}}{(Re_R)^{1/2}} I \left(\frac{x}{2R}\right)^{(\gamma-1)/\gamma} \times \left(\ln\frac{x}{2R}\right)^{1/2}\right]^{\gamma/2(\gamma+1)} \left(\frac{x}{2R}\right)^{(1-\gamma)/2(\gamma+1)}$$

and

$$\delta = [\gamma/(\gamma + 1)]^{1/2}\, (\tfrac{1}{4})^{(1-\gamma)/\gamma}\, K^{(2-\gamma)/2\gamma}$$

and

$$a^* = (4/K)^{1/2(\gamma+1)}\, (\epsilon/4)^{\gamma/2(\gamma+1)}\, (y_s/y_e)^{\gamma/(1+\gamma)}$$

over a range where

$$(1 - \gamma)/2(1 + \gamma)\ \ln|x/2R| \ll 1$$

and a/a^* close to one. If the two-dimensional continuity relation were applied to the axisymmetric case (neglecting the effect of transverse curvature and thus assuming y_e is of the order R), the resulting value for a is given in Ref. 3. This approach gives approximately the same value for a^* as the

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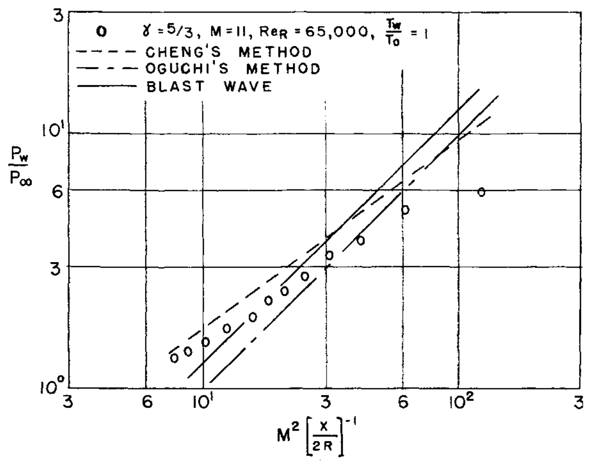

Fig. 1 Pressure distribution

forementioned method (within 3%) for both $\gamma = \frac{7}{5}$ and $\frac{5}{3}$. Once the pressure distribution is known the heat transfer can be computed in the previously indicated manner.

The forementioned theories are compared with the experimental pressure and heat transfer distributions obtained on a flat faced circular cylinder in helium at $M \sim 11$ on Figs. 1 and 2. In applying Oguchi's method a value of $a/a^* = 0.90$ was chosen. If the actual value of a were used, its deviation from the assumed value would be a maximum of $\pm 5\%$. Also examining the equation for a, it is seen that the slope of the theory on Fig. 1 would increase, but this change would be in poorer agreement with experiment than the one presented. The blast wave solution without the correction for finite freestream pressure is also presented on Fig. 1. For the extension of Cheng's theory, the drag coefficient was taken to be equal to 1.71. Examining both the pressure and heat transfer data, it is seen that only the present extension of Cheng predicts the correct variation of the data. Cheng's method was derived in the limit of small ϵ, and the present comparison is for helium with $\epsilon = \frac{1}{4}$. A first-order correction for the effect of finite ϵ will tend to raise the overall level of the predicted pressure distribution. This correction will decrease the accuracy of the theoretical pressure distribution but improve the agreement of the theory with the heat transfer results. Although the two theories are in poor agreement with the heat transfer results, they are as good as or better than various other theories that require a pressure distribution obtained from experimental data (see Ref. 3).

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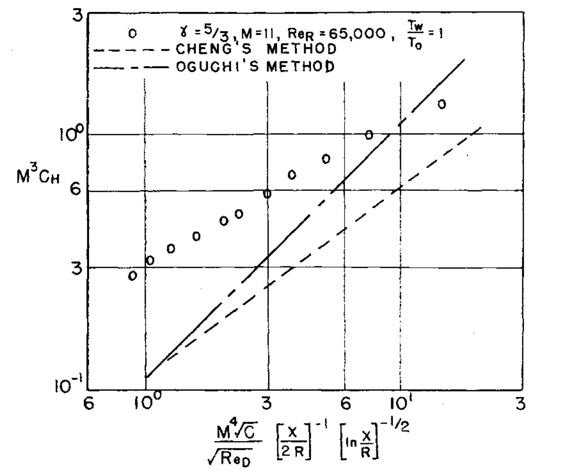

Fig. 2 Heat transfer distribution

Thermal Conductivity of Gaseous Unsymmetrical Dimethylhydrazine

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The thermal conductivity of gaseous unsymmetrical dimethylhydrazine was determined by a modified hot-wire technique employing five standard gases. Data were obtained at average temperatures of about 5°, 10°, 30°, and 35°C. All gases were run at a pressure of $\frac{1}{6}$ atm in order to overcome convection effects. Assuming linearity with temperature, the thermal conductivities were 266 × 10⁻⁷ and 316 × 10⁻⁷ cal/cm-sec-°C at 0° and 50°C, respectively.

In the hot wire method, a gas at specified temperature and pressure is admitted to a long, narrow cylinder whose axis coincides with a thin wire. This wire is electrically heated by a stable d.c. power supply to a temperature slightly higher than that of the cylinder wall. The temperature difference must be large enough to allow accurate measurement but not so great as to produce large variations in gaseous thermal conductivity between wire and wall. The resistance of the wire as a function of temperature is known, so that resistance measurements determine wire temperatures. The temperature of the cylinder wall is maintained at a preselected value by immersion in a constant temperature bath. Heat flow rate from wire to wall is calculated from the amperage and voltage drop along the test length. The wire should be thin enough to approximate semi-infinite length; this geometry produces a long constant-temperature test section.

The total heat flow rate from the constant temperature section of the heated wire to the cylinder wall is given by the equation

$$Q_t = JEI = Q_k + Q_c + Q_r \qquad (1)$$

where J is a conversion constant, E voltage drop along the test section, I amperage through the circuit, Q_k heat flow rate due to conduction, Q_c heat flow rate due to convection, and Q_r heat flow rate due to radiation.

A previous investigator[1] has reported that convection effects are minimized by orienting the test cylinder in vertical position. Findings in the present study have verified this conclusion. Convection is nearly eliminated by testing at reduced pressure; at 1 atm, 2.5% of the heat transfer was attributable to convection, whereas at $\frac{1}{6}$ atm, convective heat transfer was negligible. Likewise, heat transfer by radiation is very small if the filament area is low and if the temperature drop from filament to cylinder wall is not large.

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